Power Optimization on a Network: The effects of randomness

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***Abstract*—Consider a wireless network of transmitter-receiver pairs. The transmitters adjust their powers to maintain a particular SINR target in the presence of interference from neighboring transmitters. In this paper we analyze the optimal power vector that may achieve this target in the presence of randomness in the network. Specifically, we start from a regular grid of transmitter-receiver pairs and randomly turn-off a finite fraction of them. We apply concepts from random matrix theory to evaluate the asymptotic mean optimal power per link, as well as its variance. Our analytical results show remarkable agreement with numerically generated networks, not only in one-dimensional network arrays but also in two dimensional network geometries. Remarkably, we observe that the optimal power in random networks does not go to infinity in a continuous fashion as in regular grids. Rather, beyond a certain point, no finite power solution exists.**

I. Introduction

Transmitted power is an important resource in wireless networks and therefore power control has been crucial since the development of legacy networks. For example, the introduction of efficient power control algorithms (both closed loop and open loop), was one of the main improvements third generation CDMA-based cellular networks brought about. Several algorithms have appeared that provably allow users to obtain e.g. a minimum SINR requirement $SINR_k \geq \gamma_k$ for link k while minimizing the total power or the power per user, subject to the feasibility of this solution. [1], [2]

Of course, power optimization remains an important problem in emerging and future networks. Ad hoc networks are one such class, where substantial effort has been made to analyze their behavior, such as connectivity and transport capacity [3]. However, the majority of such work has been focused on *fixed* power transmitted and has not analyzed the potential benefits of power optimization. Power control together with scheduling becomes more important when multiple hops on the network are necessary for information to reach its destination. In a different setting, the density of WiFi networks is already high enough to create significant interference. This will also become an issue when femto-cells, a recently proposed network paradigm, will become massively deployed: due to their close proximity, neighboring femto-cells may create interference to one another [4]. As a result, all above situations are expected to benefit tremendously from power optimization. Nevertheless, little progress has been made in finding analytic estimates of the performance of random interference-limited networks when power control is applied [5].

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In this paper we apply tools from random matrix theory, [6], [7] provide an analytic estimate of the optimal power performance for a large network in the presence of both interference and randomness. We will start from an ordered network structure, in particular an equally spaced line (or square) of N transmitters, each with a receiver located in its neighborhood at a fixed distance. We will then add randomness in the network by removing each base-receiver pair with probability e. As a result, a network with roughly $N(1-\mathrm{e})$ transceiver pairs will remain, randomly located in the original grid. This sparse network is a model for a realistic cellular network, which usually tries to mimic a (usually hexagonal) symmetric network. It is also a good model for a wireless network with intermittent activity, such that at any time only a fraction ρ of the total bases are active. Although relatively straightforward, this paradigm carries all the traits of a random network and, as we shall see, behave qualitatively different from ordered networks. Also, surprisingly these results are valid for both one-dimensional and two dimensional networks (arrays and square-grids).

The paper is organized as follows. In the next section we define the network model in the absence of any randomness. In Section III we describe the network with erasures of base-receiver pairs and calculate the average minimum power per user as well as its variance using results from random matrix theory. We also discuss the relevance of these results in both one-dimensional and two dimensional geometries. Finally, in Section IV we draw some conclusions, while at the Appendix we discuss some details of the proofs.

II. Model Description

Assume a network of N base-stations on a lattice. The lattice may be one-dimensional with equal spacing ℓ between them, or two-dimensional (see Fig. 1). In the latter case, which is obviously the more realistic one, it may have any symmetry, e.g. hexagonal or triangular, but for concreteness, we will assume that the lattice is square. Each base-station is connected via a link to a single user located at a distance δ from its corresponding base-station. The (average) channel coefficient between base-station i and user j is given by

$$g_{ij} = \frac{\delta^\alpha}{(|\mathbf{m}_i - \mathbf{m}_j|^2\ell^2 + \delta^2)^{\alpha/2}} \qquad (1)$$

where $\mathbf{m}_i$ is the lattice vector of integers corresponding to base-station i. For simplicity, we have normalized the channel gains to unity for $i = j$. We also define $s = \delta/\ell$. For d-dimensional lattices ($d = 1, 2$), it can be expressed in terms of

integer linear combinations of the d basis-vectors of the lattice

$$\mathbf{m}_i = \sum_{k=1}^{d} m_k \boldsymbol{\epsilon}_k \tag{2}$$

for integer $0 \leq m_k < L$, where $N = L^d$. Since the integers m_k fully specify the position $\mathbf{m}_i$, we will drop the index i when possible. For simplicity we assume periodic boundary conditions on the lattice, hence $m_k \equiv m_k + L$. This means that the distance between any two points is taken as their minimum distance on a toroidal geometry (or a circular geometry for one dimension). This has no effect on neighboring sites or for large sizes, and, in addition, it enforces the circulant property of the matrix g_{ij}. Also, $\alpha \geq 2$ is the pathloss exponent, which signifies how fast the channel strength decays as a function of distance.

The pathloss function above is somewhat artificial in its dependence on the distances between the receiver user and the transmitting base. Technically, it is strictly correct only when each user is located vertically to the line connecting all bases (as in any horizontal line of Fig. 1), which is one possible geometry for one dimensional systems [8]. Nevertheless, it has the right behavior for $\mathbf{m}_i = \mathbf{m}_j$ as well as for $|\mathbf{m}_i - \mathbf{m}_j|\ell \gg \delta$. [1] The reason we chose to use this model will become apparent later.

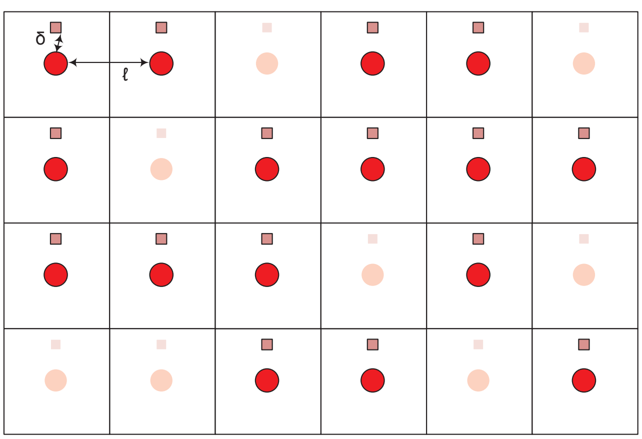

Fig. 1. Schematic figure of wireless network The circles correspond to base-stations, while the squares receiver users. The opaque squares represent base-user pair that have been "erased" (inactive).

The users are assumed to try to connect with their own nearby cells. The SINR for each connection is given by

$$SINR_k = \frac{p_k g_{kk}}{n + \sum_{j \neq k} g_{jk} p_j} \tag{3}$$

where p_k is the transmitted power from the k base to the user k and n is the thermal noise, assumed equal for all for simplicity. For the connection to be possible, a minimum value of the SINR has to be attainable i.e.

$$SINR_k \geq \gamma_0 \tag{4}$$

Therefore, each base should adjust its power to meet this criterion. As a result, the following set of equations should be simultaneously met

$$n^{-1}\gamma_0^{-1} g_{kk} p_k - \sum_{j \neq k} n^{-1} g_{jk} p_j \geq 1 \tag{5}$$

The above equations constitute a set of linear (planar) constraints on the powers. The minimum total power is reached at the apex of the conical section of allowed powers, given by equality of all constraints above. Defining the matrix $\mathbf{M}$ with elements M_{ij} as

$$M_{ij} = \begin{cases} n^{-1}\gamma_0^{-1} g_{ii} & i = j \\ -n^{-1} g_{ji} & i \neq j \end{cases} \tag{6}$$

we may write the above equation as

$$\mathbf{P} = \mathbf{M}^{-1}\mathbf{J} \tag{7}$$

where $\mathbf{P}$ is the vector of powers that satisfies the equality constraint above and $\mathbf{J} = [1, 1, \ldots, 1]^T$. As a result, the minimum average power is given by

$$p_{ave} = \frac{1}{N}\mathbf{J}^T\mathbf{M}^{-1}\mathbf{J} \tag{8}$$

Clearly, for the inverse $\mathbf{M}$ to be well-defined, all its eigenvalues have to be positive.

Now, due to the circulant structure of the matrix, its eigenvectors will be equal to $\mathbf{v_q}$, which at position $\mathbf{m}$ has the value

$$\mathbf{v_{q,m}} = \frac{e^{i\mathbf{q}\cdot\mathbf{m}}}{L} \tag{9}$$

where the vector $\mathbf{q}$ resides in the fundamental cell of the reciprocal lattice of the lattice $\mathbf{m}$

$$\mathbf{q} = \frac{2\pi}{L}\sum_{s=1}^{d} k_s \hat{\boldsymbol{\epsilon}}_s \tag{10}$$

with $0 \leq k_s < L$ [10]. The eigenvalue corresponding to the vector $\mathbf{q}$ is simply the Fourier transform of any line of the matrix $\mathbf{M}$

$$\begin{aligned} \lambda(\mathbf{q}) &= \sum_{\mathbf{m}} M_{0,\mathbf{m}} e^{i\mathbf{q}\cdot\mathbf{m}} \\ &= n^{-1}\left(\gamma_0^{-1} - \sum_{\mathbf{m}\neq 0} \frac{s^\alpha e^{i\mathbf{q}\cdot\mathbf{m}}}{(|\mathbf{m}|^2 + s^2)^{\alpha/2}}\right) \end{aligned} \tag{11}$$

It is important to note that these eigenvalues are real because the matrix $\mathbf{M}$ is real and symmetric, which is the reason for choosing (1). Finally, since the vector $\mathbf{J}$ is proportional to the $\mathbf{q} = 0$ eigenvector of $\mathbf{M}$ we conclude that

$$p_{ave} = \frac{1}{\lambda_0} = \frac{n\gamma_0}{1 - \gamma_0 \sum_{\mathbf{m}\neq 0} \frac{s^\alpha}{(|\mathbf{m}|^2 + s^2)^{\alpha/2}}} \tag{12}$$

[1] In fact, we believe this accuracy of this approximation is better than that of a pathloss exponent $\alpha > 2$ as a free variable, whose value comes out of curve fitting of large amounts of data with sizable errors [9].

where $\lambda_0 = \lambda(\mathbf{q} = 0)$. This solution corresponds to all bases transmitting with the same power. The condition for finite total power in the case of no erasures is simply

$$\gamma_0 \sum_{\mathbf{m} \neq 0} \frac{\delta^\alpha}{(|\mathbf{m}|^2 \ell^2 + \delta^2)^{\alpha/2}} < 1 \tag{13}$$

III. SPARSE NETWORK

The above equations are valid in the absence of erasures of base-stations. Erasures may take place in two cases. First, perhaps in a particular location a base station may not have been erected. Therefore, no mobile is expected to receive any data in that cell. Second, in a cell with a few users present, once in a while all data buffers for all connected users may be empty. Hence the corresponding base-station will be silent. In principle, it may still be transmitting a pilot signal, in which case the required SINR will be much smaller. For simplicity, we will assume that the connection to its cell-user(s) will be off. As a result, we are faced with a situation, in which, each row and column, with probability e has zero entries. Thus the matrix we now need to analyze is

$$\mathbf{M}_\mathrm{e} = \mathbf{EME} \tag{14}$$

where $\mathbf{E}$ is a diagonal random matrix with independent diagonal entries e_i on the ith entry, with

$$P[e_i = 0] = \mathrm{e} \tag{15}$$
$$P[e_i = 1] = 1 - \mathrm{e}$$

The corresponding equation for the power vector becomes

$$\mathbf{EMEP} = \mathbf{EJ} \tag{16}$$

where we have made sure to also erase the appropriate entries from $\mathbf{J}$ on the left-hand-side by multiplying with $\mathbf{E}$. However, simply inverting the matrix $\mathbf{M}_\mathrm{e}$ is problematic, since it has approximately $N\mathrm{e}$ zero columns and rows and corresponding zero eigenvalues. Instead, we regularize the matrix by adding a small positive constant on the diagonal, namely

$$\mathbf{P} = \left[\mathbf{EME} + \epsilon \mathbf{I}\right]^{-1} \mathbf{EJ} \tag{17}$$

To obtain the average power as before we multiply on the left with $\mathbf{J}^\dagger \mathbf{E}$ to collect all the components with non-zero power. Thus we have

$$p_{ave} = \frac{1}{1-\mathrm{e}} \lim_{\epsilon \to 0^+} \frac{1}{N} \mathbf{J}^\dagger \mathbf{E} \left[\mathbf{EME} + \epsilon \mathbf{I}\right]^{-1} \mathbf{EJ} \tag{18}$$

We will state the result below, and defer the proof to Appendix A.

Theorem 1 (Average Minimum Power)**.** *Let β be the solution of the equation*

$$\mathrm{e} = \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{\beta}{\beta + \lambda(\mathbf{q})} \tag{19}$$

where $\lambda(\mathbf{q})$ is the $\mathbf{q}$ eigenvalue of the matrix bM (11) and the integral is over the fundamental cell of the reciprocal lattice of the system. Then the average power per base-station is given by the equation

$$p_{ave} = \frac{1}{1-\mathrm{e}} \frac{1}{\beta + \lambda_0} \tag{20}$$

Remark 1.1. In the appendix the above theorem is proved straightforwardly for one-dimensional lattices using results from [6]. We will argue that this is valid for two-dimensional lattices and for all feasible γ_0 through simulations and defer the complete proof for a longer version of the work.

It is easy to see that the above result reduces to (12) in the limit that $\mathrm{e} \to 0$. Indeed, when $\lambda_0 > 0$, the right-hand-side of (19) is zero only when $\beta = 0$. Hence (12) follows. We can also show that for $\mathrm{e} \to 1$ the average power simply takes its non-interference value

$$p_{ave} \approx \gamma_0 n \tag{21}$$

In the presence of erasures, the maximum SINR target that is supported in the system is reduced to

$$\gamma_\mathrm{e} = \frac{\gamma_0}{1 + n\beta_\mathrm{e}\gamma_0} \tag{22}$$

where β_e is the solution of (19) and γ_0 is given by the equality of (13).

In addition to the mean power, we can also obtain the variance of the powers in the network. Again the proof is deferred for Appendix A.

Theorem 2 (Variance of Minimum Power)**.** *The variance of the powers of the individual bases in the network is given by*

$$Var(\mathbf{P}) = \frac{1}{1-\mathrm{e}} \frac{1}{(\beta+\lambda_0)^2} \frac{\int \frac{d\mathbf{q}}{(2\pi)^d} \frac{\beta}{(\beta+\lambda(\mathbf{q}))^2}}{\int \frac{d\mathbf{q}}{(2\pi)^d} \frac{\lambda(\mathbf{q})}{(\beta+\lambda(\mathbf{q}))^2}} \tag{23}$$

A. Analysis of one dimensional array

We will start by analyzing the results for the simpler one-dimensional case, where the base-stations are located in a ring. The numerical results are plotted in Figures 2, 3.

First, we see that the average power obviously is a increasing function of γ_0 and a decreasing function of e.

Second, we see, both analytically and numerically, that the average power is finite for a range of γ_0 above the value for which $\lambda_0 = 0$, which is the value of γ_0 for which the $\mathrm{e} = 0$ solution diverges. However, this comes at a cost: For such γ_0 for which $\lambda_0 < 0$ a second solution to the fixed point equation appears (plotted, for the case of $\alpha = 4$ in Fig. 3 in green). This value of p_{ave} is unstable however, having a *negative* variance (23). This second solution of p_{ave} persists until it merges with the stable value at a critical value of γ_0. However, at that point the variance of the stable solution diverges. Beyond this point no finite average power can be supported for the given value of e. Thus we see a discontinuous transition at that point, from a finite average minimum power to an infinite value. This is because beyond this point, at least one power has to be infinite. In fact, the finite value of p_{ave} all the way to the critical value suggests that at most a vanishing fraction of bases have unbounded powers.

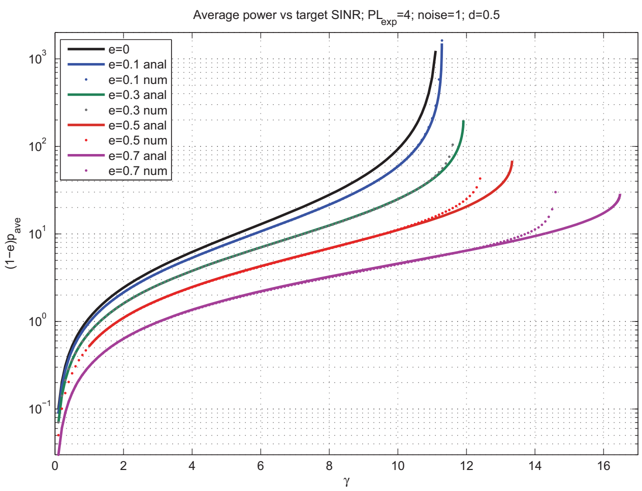

Fig. 2. Plot of average power as a function of the target SINR for various values of e. The red curves correspond to the numerically generated values. The path-loss exponent is set to $\alpha = 4$, while the noise level $n = 1$ and the ratio $s = \delta/\ell = 0.5$

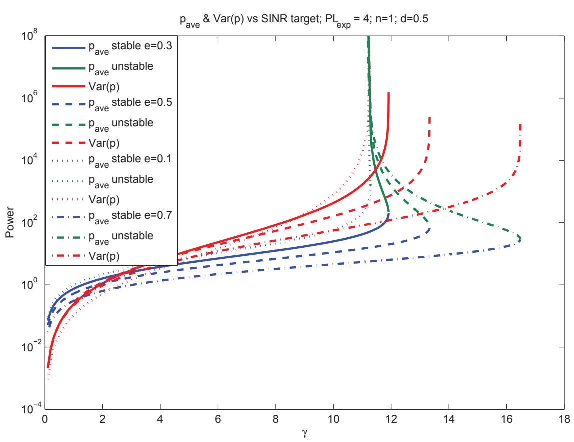

Fig. 3. Plot of average power as a function of the target SINR for various values of e for the parameter values of Fig. 2. In addition to the curves plotted also in Fig. 2 (blue) we plot the curves corresponding to the unstable solutions of (19) (green) and the variance of the power for the stable solution (red).

Another interesting result from the numerical analysis is that beyond the value of γ where $\lambda_0 = 0$ the behavior becomes sample dependent. Thus the value at which the power becomes infinite varies somewhat from sample to sample (as well as the whole curve beyond the value where $\lambda_0 = 0$). As we shall show in the Appendix, the end point of this curve has to do with the largest eigenvalue of the matrix $\mathbf{EME}$. For finite but large N it is known for similar random matrix systems [11] that although the eigenvalues of the matrix converge fast to their asymptotic spectrum, the largest eigenvalue fluctuates much more outside the support of the asymptotic spectrum. Hence, the theoretical curve seems to be the upper limit, beyond which no finite power can be supported. The numerical curves in Fig. 2, especially for the larger values of e have been picked after running several simulations and picking the

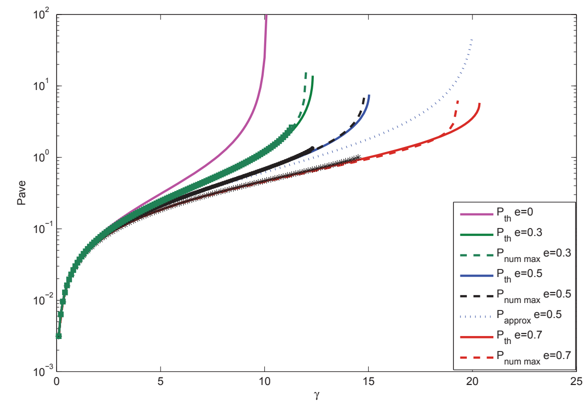

Fig. 4. Plot of average power as a function of the target SINR for various values e for a two-dimensional square grid of base-stations. The path-loss exponent is $\alpha = 5$. The curves have the same meaning as in the previous figure.

one with the largest critical value. How can we explain this behavior? For example, if a region happens to have a large concentration of bases it may create a situation where the power of a particular base diverges. Such a happenstance may be less probable if each base is affected by a "mean-field" of all bases, including far away ones.

B. Analysis of two-dimensional grid

It is remarkable that these results also hold for two-dimensional grids, even though we have not provided a proof in this short paper version. This can be seen in Fig. 4 where the average optimum power for a square grid 50×50 of base-user pairs is plotted as a function of γ for different densities of erasures e. We plot the analytic curve as well as the numerical average over approximately 500 realizations (solid line) and the curve for the realization that has lasted to the largest γ before becoming unfeasible (dashed). We once again observe the same behavior. The "average" curve stops at some point after the value of γ for which $\lambda_0(\gamma) = 0$ while the dashed lines go up to close to the analytic curve. In Fig. 4 we also plot for comparison a "mean-field" curve, which assumes that the interference is spread evenly over all neighboring sites, such that

$$\gamma = \frac{p g_{ii}}{n + \sum_{j \neq i} g_{ij}(1 - \mathrm{e})p} \tag{24}$$

It becomes clear that while at small γ it behaves decently, it significantly overshoots the singular point.

IV. Discussion

In this paper we have studied the optimal power vector that achieves an SINR target criterion in a wireless network setting where both randomness and interference are relevant. We have applied methods from Random Matrix Theory to obtain the average optimum power and its variance. Our numerical results show remarkable agreement with predicted analytical calculations, both for one- and two-dimensional network geometries. We observe that in the presence of randomness the optimal

power does not go to infinity in a continuous fashion. Rather, beyond a certain point, no finite power solution exists. We also numerically observe that beyond the point where the pure system has finite optimal power, real random systems become unstable, making finite power unfeasible for increasing SINR targets at different points. The ones that remain finite reach our analytical limit. This suggests that randomness creates sample to sample fluctuations, which make local clusters of cells and hence the whole network unfeasible to operate at that target SINR. Therefore, it does not make sense to look for globally optimal power vectors beyond a certain network size. Instead sacrificing the connection of a few "bad" bases by adding some outage may increase the overall behavior dramatically.

APPENDIX A
PROOF OF THEOREMS 1 AND 2

Both proofs will be assume one-dimensional networks. The case of $d = 2$ is deferred to a later publication.

Proof of Theorem 1: We start by defining the matrix $\mathbf{A}$ as

$$\mathbf{A} = \mathbf{E}\mathbf{F} \tag{25}$$

where $\mathbf{F}$ is the Fourier matrix that diagonalizes $\mathbf{M} = \mathbf{F}^\dagger \mathbf{\Lambda} \mathbf{F}$ and denote its column vectors by $\mathbf{a}(\mathbf{q})$ and, in particular, $\mathbf{a}(0) = N^{-1/2}\mathbf{E}\mathbf{J} = \mathbf{a}_0$. As a result, (18) may be rewritten as

$$\mathbf{a}_0^\dagger \left[\lambda_0 \mathbf{a}_0 \mathbf{a}_0^\dagger + \mathbf{A}\mathbf{\Lambda}_0\mathbf{A}^\dagger + \epsilon \mathbf{I}\right]^{-1} \mathbf{a}_0 \tag{26}$$

where $\mathbf{\Lambda}_0$ is the diagonal matrix of eigenvalues of $\mathbf{M}$ with the one corresponding to $\mathbf{q} = 0$ (i.e. λ_0) set to zero. After the application of the matrix inversion lemma we obtain

$$p_{ave} = \frac{1}{1 - \mathrm{e}} \lim_{\epsilon \to 0^+} \frac{1}{\beta(\epsilon) + \lambda_0} \tag{27}$$

where

$$\beta(\epsilon)^{-1} = \mathbf{a}_0^\dagger \left[\mathbf{A}\mathbf{\Lambda}_0\mathbf{A}^\dagger + \epsilon \mathbf{I}\right]^{-1} \mathbf{a}_0 \tag{28}$$

In [7] it is shown that the above quantity takes (a.s.) a deterministic value in the large N limit, which is the solution of the following equation

$$\frac{\mathrm{e}}{1 - \frac{\epsilon}{\beta(\epsilon)}} = \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{\beta(\epsilon)}{\beta(\epsilon) + \lambda(\mathbf{q})} \tag{29}$$

Taking the limit $\epsilon \to 0^+$ in the above and setting $\beta(0) = \beta$ concludes the proof. ■

We now turn to finding the variance of the powers

Proof of Theorem 2: We start by noting that the normalized second moment of the power vector $\mathbf{P}$ is given by

$$\begin{aligned} \frac{\mathbf{P}^\dagger\mathbf{P}}{N} &= \frac{1}{1 - \mathrm{e}} \lim_{\epsilon \to 0^+} \mathbf{a}_0^\dagger \left[\mathbf{EME} + \epsilon \mathbf{I}\right]^{-2} \mathbf{a}_0 \\ &= -\frac{1}{1 - \mathrm{e}} \lim_{\epsilon \to 0^+} \frac{\partial}{\partial \epsilon} \mathbf{a}_0^\dagger \left[\mathbf{EME} + \epsilon \mathbf{I}\right]^{-2} \mathbf{a}_0 \\ &= \frac{1}{1 - \mathrm{e}} \lim_{\epsilon \to 0^+} \frac{\beta'(\epsilon)}{\left(\beta(\epsilon) + \lambda_0\right)^2} \end{aligned} \tag{30}$$

where $\beta'(\epsilon)$, the derivative of $\beta(\epsilon)$ with respect to ϵ can be evaluated directly from (29). Subtracting off p_{ave}^2 concludes the proof. ■

It is interesting to point out that the condition that the denominator in (23) vanishes, which signifies the end of the feasibility of a solution, is equivalent that the largest edge of the asymptotic spectrum of the matrix $\mathbf{EME}$ is zero and hence β cannot admit a real solution. To show this we start from the realization the fact that $\mathbf{M}$ and $\mathbf{E}$ are asymptotically free [7]. Then defining the Cauchy transform as

$$F(z) = \frac{\mathrm{e}}{z} - \frac{\eta(-z^{-1})}{z} \tag{31}$$

where η is the η transform of the matrix [12]. Taking advantage of asymptotic freedom [7], [12] we can show that

$$\mathrm{e} = \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{\mathrm{e}/F(z)}{\lambda(\mathbf{q}) - z + \mathrm{e}/F(z)} \tag{32}$$

Identifying $\mathrm{e}/F(0)$ as β we see that if $z = 0$ is not part of the asymptotic spectrum, β has a real solution in the above equation, which coincides with (19). On the other hand, when $z = 0$ is inside the support of the spectrum, β will also have an imaginary part. At the edge of the spectrum the condition $ImF(z_{edge}) = 0$ implies

$$\int \frac{d\mathbf{q}}{(2\pi)^d} \frac{\lambda(\mathbf{q}) - z}{(\lambda(\mathbf{q}) - z + \beta)^2} = 0 \tag{33}$$

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